**Discrete age-structured SEIR epidemic model with applications to measles vaccination strategies**

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**Mathematical Model**

**An application to vaccination strategies for measles**

Measles is a disease that can be prevented with a vaccine. Given as part of the measles-mumps-rubella (MMR) vaccine, the measles vaccine typically requires two doses. The first dose of measles vaccine is commonly given to infants at 6 months of age and the second dose is administered at 12 months of age. The efficacy of two doses of measles vaccine range between 93% to 99%. In South Africa, vaccine coverage requires a maximum of 95% or higher in order to be sustained with both doses administered per person.

In this section, we develop a two-dose vaccination version with two age corporations to observe the vaccination strategies for measles epidemics.

**Measles vaccination model**

Measles can be prevented with the MMR vaccine. The CDC recommends that children receive the MMR vaccine twice. The first dose is 6 months old and the second dose is 12 months old [19]. One dose of MMR vaccine is 93% effective against measles while two doses of MMR vaccine are 97% effective against measles. In South Africa, vaccine coverage of children at 12 months old age is averaged 71.1%, while second dose averaged 68.8% between year 2012 to 2017. The coverage of the second dose increased to 76.4% in 2018 [].

We subdivide the host population into two age groups, considering age-specific differences in vaccination schedules, mortality, and contact patterns.

Figure 1: Transfer diagram for a vaccination model with two age group

The model structure is shown in the transmission diagram in Figure 1. Two doses of measles vaccine were incorporated: MMR1 for age group 1 (6 months), MMR2 for age group 2 (12 months). The model is described by the following system of differential equations.

Differential equations for age group 1:

Differential equations for age group 2:

The model parameters are shown in Table 1 along with their description and parameters. Specifically, and are the vaccination rates of MMR1 and MMR2, respectively, and are the efficacy of MMR1 and MMR2, respectively, and and are the effective coverage of MMR1 and MMR2, respectively.

To incorporate vaccination, assume a proportion, , of 6-month-old into the population are vaccinated (and thus immune to infection). Those that are vaccinated will avoid the susceptible class and go straight to the recovered class, whereas those that are unvaccinated will go into the susceptible class as before. If is the proportion vaccinated, then is the proportion left unvaccinated.

The transmission coefficient βkj between Sk and Ij is decomposed into two factors βkj = ckj where βk k is the probability of transmission for an average contact between a susceptible individual in age group k Sk, and ckj is the mean. Number of contacts between people in age group and people in age group k. Note that ckj and cjk are not the same and the contact matrix (cjj) may not be symmetric because of different ages.

Influx susceptible individuals are recruited by the rate of Λk. Exposed individuals move to the infectious class at a rate of an age group of ϵk. Infectious individuals move to the recovered compartment at a rate of an age group of γk. Individuals are aging at a rate αk. Natural fatality rate of an age group is represented by dk, while case fatality of a age group is represented by a rate of μk.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Parameters | Values/Range | Unit | Description | Ref |
|  | 500 |  | Influx of susceptible | [] |
|  |  |  | The natural mortality rate of age group k | [] |
|  |  |  | Aging rate of age group k | [] |
|  |  |  | Recovery rate of age group k | [] |
|  | 0.72 |  | Exposed rate of age group | [32] |
|  |  |  | Case mortality rate of age group k | [] |
|  | 0.717 |  | Vaccination coverage of Measles vaccine | [28] |
|  | 0.764 |  | Vaccination coverage of Measles vaccine | [28] |
|  | 0.93 |  | Efficacy of MMR1 | [29] |
|  | 0.95 |  | Efficacy of MMR2 | [29] |
|  |  |  | Probability of transmission per contact for age group 1 | [] |
|  |  |  | Probability of transmission per contact for age group 2 | [] |
|  |  |  |  | [] |
|  |  |  | Average number of contacts from age group j to age group k | [] |

Table 1: Parameters and their estimated values for model

**Parameters estimation and model calibration**

As indicated in Table 1, values of some parameters and initial values of state variables in model are estimated directly from published data. Other parameter values, especially those of the probability of transmission per contact and the recovery rate from measles for each age group, are estimated by fitting the model outcomes to measles data using the nonlinear least squares method [30]. The measles data used for model fitting include the reported annual incidence and age specific incidence of measles in India from 2015 to 2020 [31]. The values of measles case mortality ratio are and . By the end of 2020, the values of , are the actual vaccination rates published by NICD.

**Immune profile analysis**

In our model, we generated the measles immune profile analysis for total population and for different age group. The endemic level of measles vaccination strategies in South Africa, namely, a single-dose vaccine at 6 months old (age group 1) and the second dose vaccine at 12 months old (age group 1) during years 2015-2020. In South Africa, vaccination coverage of children under 1 year averaged 71.1% , whilst measles second dose vaccination coverage is 76.4% [28]. The efficacy of two doses of measles vaccine range from to 93-99%. We therefore assume that the efficacy of the first dose is 93% and for the second dose is 95% [29].

**Effect of increasing measles and improving vaccination coverage**

Vaccination is critical to sustaining and increasing vaccination coverage rates and preventing outbreaks of measles vaccine preventable disease. The strong enforcement may help promote higher rates of vaccination coverage along with complementary actions such as monitoring VPD cases. The vaccination coverages of single-dose (MMR 1) and the second dose (MMR 2) should be both increased to 95% . In South Africa, the efficacy of MMR 1 can reaches 99% when administered to children 6 months old.

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